Magnetic Field and Current Sensors with Robust Spun Fiber

Viatcheslav G. Izraelian
IVG Fiber Development, Toronto, Canada, ivgfiber@home.com

ABSTRACT

The concept of robust sensor of current and magnetic field was under consideration based on the use of polarization interferometer scheme. The scheme utilized broadband lightsource and singlemode fiber with significant intrinsic birefringence comparable with the period of birefringence axes revolution, their ratio N (number of revolutions on a local beatlength) being the main parameter both for sensor robustness and sensitivity determination. The external forces-originated perturbations were taken into account in the “method of averaging” approximation and an optimal condition for number N was determined to suppress perturbations’ influence. Even for perturbed case the polarization retardation in such a fiber was found not accumulating but varying periodically. Conditions for these varying terms (responsible for sensor’s scale factor instability) to vanish due to source bandwidth averaging were evaluated.

Keywords: fiber, optics, sensor, current, magnetic

1. INTRODUCTION

In past several years fiber optic current sensor was under consideration\textsuperscript{1-3} due to its obvious advantages: electrical and environmental safety, low mass and volume and low utilised power. Magnetic field sensors utilizing low-birefringence (LB) fibers obtained by spun drawing technique proved these fibers to be effective in accumulating Faraday rotation along the fiber axes, so the signal sensitivity could be improved beyond acceptable figures simply by fiber length increase. However LB fibers appeared to be very sensitive to external exposures: bending, squeezing, temperature variations etc. That means that to achieve good accuracy for "field circumstances" (particularly in a broad temperature range) one have either to protect the optical scheme from external parameters changes or to install additional sensors to compensate correspondent errors. This leads to significant increase of sensor complexity making this device to be less competitive in comparison with conventional amperemeters. It seems to be relevant to suggest some option of sensitive fiber that is free of this disadvantage.

2. SENSOR ACCURACY CHARACTERISTICS

Sensitive fiber properties can be described\textsuperscript{4} by fiber piece’s Jones matrix $\mathbf{J}$ defined through:

$$\mathbf{E}_{\text{out}} = \mathbf{J} \mathbf{E}_{\text{in}},$$

where $\mathbf{E}_{\text{out}}, \mathbf{E}_{\text{in}}$ are two-dimensional vectors, their components being x and y polarizations amplitudes in laboratory frame. Then the Jones matrix of whole piece with length $L$ can be obtained as a solution (for $z=L$) of coupled-mode (polarization) equation:

$$i \frac{d\mathbf{J}(z,m)}{dz} = \left[ \mathbf{V}(z) + i \mathbf{m} \mathbf{\Gamma} \right] \mathbf{J}(z,m),$$

\begin{align*}
&\begin{bmatrix}
    B(z)\cos[2\alpha(z)] & B(z)\sin[2\alpha(z)] \\
    B(z)\sin[2\alpha(z)] & -B(z)\cos[2\alpha(z)]
\end{bmatrix} \\
&\begin{bmatrix}
    0 & -1 \\
    1 & 0
\end{bmatrix}
\end{align*}

$\mathbf{J}(0,m)=\mathbf{E}; \mathbf{V}=(1/2)\begin{bmatrix}
    B(z)\cos[2\alpha(z)] & B(z)\sin[2\alpha(z)] \\
    B(z)\sin[2\alpha(z)] & -B(z)\cos[2\alpha(z)]
\end{bmatrix}; \mathbf{\Gamma} = \begin{bmatrix}
    1 & 0 \\
    0 & 1
\end{bmatrix}; \mathbf{E} = \begin{bmatrix}
    0 & 1
\end{bmatrix}$
where $\mathbf{B}(z)$ is fiber local birefringence at a point with $z$ coordinate: $\mathbf{B}(z) = \frac{2\pi}{L_b} = \frac{2\pi \Delta n}{\lambda}$, $L_b$ is a local beatlength, $\Delta n = n_x - n_y$ – refractive indexes difference, $\lambda$ - wavelength. $\alpha(z)$ is the angle of birefringence axes orientation in laboratory frame and $m$ is magnetic rotation coefficient being proportional to Verde constant and magnetic field amplitude.

We can consider the sensitive robust fiber in “double-pass” optical scheme (Fig.1):

![Fig.1](image)

Light from laser 1 obtains 100% polarization passing polarizer 2 and after passing beam-splitter 3 it is launched into sensitive fiber 4 winded around wire with current 6 (or other magnetic field source). After reflecting from the other fiber end with mirror 5 light propagates back through the fiber and then after passing beam splitter 3 again its state of polarization is determined by processing system consisting of polarization beam splitter 7 and two detectors 8 and 9. This scheme is very regular and simple and it is chosen mainly to have more definite performance for fiber features demonstration.

For this optical configuration one can assume that total Jones matrix $\mathbf{T}$ of double-passed system:

$$\mathbf{T}(L,m) = \mathbf{J}'(L,-m) \mathbf{J}(L,m)$$

(2)

where asterisk ' means matrix transposition and minus sign stands for magnetic field anti-symmetry for counterpropagation. Assuming that matrix $\mathbf{J}_0(z) = \mathbf{J}(z,0)$ is the solution of “zero magnetic field” equation:

$$i \frac{d}{dz} \mathbf{J}_0(z) = \mathbf{V} \mathbf{J}_0(z) = \frac{1}{2} \left[ \begin{array}{cc} \mathbf{B}(z) \cos[2\alpha(z)] & \mathbf{B}(z) \sin[2\alpha(z)] \\ \mathbf{B}(z) \sin[2\alpha(z)] & -\mathbf{B}(z) \cos[2\alpha(z)] \end{array} \right] \mathbf{J}_0$$

(3)

we get in the first order of small magnetic rotation parameter $m$:

$$\mathbf{T}(L,m) = \mathbf{S}(L) + mL \mathbf{A}(L)$$

(4)

$$\mathbf{S}(z) = \mathbf{J}_0^*(z) \mathbf{J}_0(z); \quad \mathbf{A}(L) = \mathbf{S}(L) \Gamma s + s \Gamma \mathbf{S}(L); \quad s = (1/L) \int dz \mathbf{S}(z)$$
When deriving (4) we used perturbation theory approximation for magnetic field-dependent term, which works well unless \( mL \) exceeds 1, symmetry conditions for matrices \( S \) and \( s \) (\( S' = S, s' = s \)) as well as the condition: \( X^T X = \det X \Gamma \) that takes place for any \( X \) (\( \det \)... stands for matrix determinant). The expression (4) can be simplified: as soon as \( A(L) \) is antisymmetrical \( (A'(L) = -A(L)) \) it can be expressed as: \( A(L) = a \Gamma \), and scalar coefficient \( a \) can be determined from:

\[
a = - \frac{1}{2} \text{Tr} \{ A(L) \} = - \text{Tr} \{ S(L) \Gamma s \} = \text{Tr} \{ S(L) \det s^{-1} \} = \text{Tr} \{ S(L) \} \text{Tr} \{ s \} - \text{Tr} \{ S(L) s \} \quad (5)
\]

Here \( \text{Tr}\{\ldots\} \) means matrix trace and we used that \( \Gamma^2 = -E \) and well-known matrix theorem:

\[
X^2 - \text{Tr} \{X\} X + \det X E = 0.
\]

As soon as we determined finally the transmission matrix \( T \) for the entire lightpath in fiber piece as:

\[
T = T(L,m) = S + a mL \Gamma
\]

we can derive the expression for sensor’s output signal on each (n-th) photodetector:

\[
I_n \sim <E_{\text{out}}^+ E_{\text{out}}> = \text{Tr} < E_{\text{out}} E_{\text{out}}^+ > = \text{Tr} < G_n T E_{\text{in}} E_{\text{in}}^+ T^+ G_n^+ > \quad (6)
\]

Here brackets \( <\ldots> \) stand for lightsource linewidth averaging, \(^+\) means hermitian conjugation and \( G_n \) \( (n = 1,2 \text{–detector number}) \) describes polarization optical path from fiber output to detector.

If we define initial light’s coherence matrix \( \rho \): \( \rho = E_{\text{in}} E_{\text{in}}^+ \) and “matrix of observation” \( \sigma \) as:

\[
\sigma = G_1^+ G_1 - G_2^+ G_2 \quad (\text{assuming that detectors operate in differential scheme}),
\]

we get (a differential scheme output signal \( I_d \) in the first order of \( mL \):

\[
I_d \sim \text{Tr} < T \rho T^+ \sigma > = \text{Tr} < S \rho S^+ \sigma > + mL < a \text{Tr} \{S \rho \Gamma \sigma - S^+ \sigma \Gamma \rho\} > \quad (7)
\]

There is a number of ways to optimize the signal parameters taking into account sensor’s accuracy characteristics, here we consider one possible configuration with linear input polarizer and axes of output polarization beamsplitter with 45 degrees orientation to input. Then, assuming matrix \( S \) to be unitary (no dichroism), output signal is:

\[
I_d \sim 2 \text{Tr} \{ \rho \} < \text{Im}(S_{xx}) \text{Im}(S_{xy}) > + mL < a \text{Tr} \{S \} > \quad (8)
\]

Here \( S_{xx}, S_{xy} \) are matrix elements of transmission matrix \( S \), depending on fiber features.

Sensor sensitivity is determined by second term in (8), which depends linearly on magnetic field compared with unavoidable noises, such as the shot-noise, caused by total light intensity fluctuations being proportional to \( (\text{Tr} \{\rho\})^{1/2} \) or lightsource excess noise being proportional directly to \( \text{Tr}\{\rho\} \), or processing system noise being intensity-independent. For all these cases fiber features-dependent term \( F_s = (1/4) <a \text{Tr}\{S\}> \) affects as a factor, its mean value being responsible for sensitivity and its external fields-caused (for example: temperature dependent) variations – for scale-factor stability. The changes in \( F_s \) thus form a multiplicative error.
For “zero-birefringence” fiber case its Jones matrix is matrix unity: $J_0 = E$, so $S = E$ and of course “spatially-averaged” matrix $s = E$. Then for that ideal case $a = \text{Tr}\{S(L)\} \text{Tr}\{s\} - \text{Tr}\{S(L) s\} = 2$, $\text{Tr}\{S(L)\} = 2$ and $F_s = 1$. For real fiber case $F_s < 1$. For example, for High-birefringence fibers spatially averaged matrix $s = 0$ and $F_s = 0$. For regular (communicational) fiber with random spatial distribution of birefringence axes angles one can expect non-zero factor $F_s$ with obvious square-root-of-length decrease. For these fibers however non-averaged matrix $S(L)$ varies strongly with external temperature, pressure etc so the scale factor stability for that case is low as well as accuracy.

The first term in (8) forms an additive error or “zero-drift” that is responsible for sensor’s accuracy. So we can define $F_a$ as: $F_a = < \text{Im}(S_{xx}) \text{Im}(S_{xy}) >$.

### 3. SPUN FIBER FEATURES

For any fiber birefringence components of differential Jones matrix in (1) and (3) are the superposition terms caused by multiple birefringence sources, either intrinsic or induced by external forces: residual intrinsic birefringence in the preform, coating influence, bending, squeezing at holders etc. so $B$ and $\alpha$ can be expressed as:

$$B \cos 2\alpha = \sum B_k \cos 2\alpha_k, \quad B \sin 2\alpha = \sum B_k \sin 2\alpha_k \quad (9)$$

Representing $V$ as: $V = H + W$:

$$i \frac{d J_0(z)}{dz} = (H + W) J_0(z) \quad (10),$$

where $H$ is the regular part:

$$H_{xx}(z) = -H_{yy}(z) = \frac{1}{2} B_i \cos[2(2\pi z/L_p + \alpha_i)]$$

$$H_{xy}(z) = -H_{yx}(z) = \frac{1}{2} B_i \sin[2(2\pi z/L_p + \alpha_i)] \quad (11)$$

$B_i$ is constant intrinsic birefringence, $L_p$ - period of its axes rotation and $\alpha_i$ is an initial angle at $z = 0$.

If $G$ is a matrix solution of (10) when $V = H$ ($W = 0$), then the $J_0$ could be obtained as a matrix product $J_0 = GX$, where $X$ is a solution of:

$$i \frac{dX}{dz} = (G^{-1}W G) X \quad (12)$$

For spun fibers, obtained by drawing technique based on anisotropic preform rotation, the regular part of interpolarisational coupling is determined by (10) with $V=H$, the residual part ($W$) being assumed to be a perturbation. Then the solution of correspondent equation:

$$i \frac{dG}{dz} = HG \quad (13)$$

can be obtained as: $G = R(2\pi z/L_p + \alpha_i) Y R(-\alpha_i)$, where $R$ is a rotation matrix:

$$R(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}, \quad Y$$ in its turn satisfies the equation:
\[ i \frac{dY}{dZ} = \begin{bmatrix} \pi/L_b - 2i \pi/L_p \\ 2i \pi/L_p & \pi/L_b \end{bmatrix} Y \]  

(14)

Equation (14) has an exact solution due to the fact that all matrix components are constant:

\[ Y = \begin{bmatrix} \pi/L_b - 2i \pi/L_p \\ 2i \pi/L_p & -\pi/L_b \end{bmatrix} \begin{bmatrix} \cos(\kappa z) E - i \sin(\kappa z)/\kappa \\ (\pi/L_p)^2 - (\pi/L_b)^2 \end{bmatrix} \]  

(15)

\[ \kappa = \pi \left( L_p^2 + 4L_b^2 \right)^{1/2} / (L_pL_b) \]

If the linear birefringence in preform is not zero: \( B = 2\pi/L_b > 0 \) then we can introduce fiber retardation \( \delta \), which does not accumulate along the fiber but oscillates with the period of \( 2\pi/\kappa \):

\[ \cos 2\delta = (1/2) \text{Tr}\{G'G\} = 1 - 2(1 + 4L_b^2 / L_p^2)^{-1} \sin^2 \{\pi z (L_p^2 + 4L_b^2)^{1/2} / (L_pL_b)\} \]

\[ \sin \delta = (1 + 4N^2)^{-1/2} \sin \{(1 + N^{-2}/4)^{1/2} \pi z / L_p\}, \quad N = L_b / L_p \]  

(16)

For this case matrix \( G \) can be expressed as:

\[ G = R(\theta) \begin{bmatrix} \exp(i\delta) & 0 \\ 0 & \exp(-i\delta) \end{bmatrix} R(-\theta) \]  

(17)

For double-pass this fiber piece behaves itself as a linear phase plate with phase retardation determined by (16), this retardation \( \delta \) never exceeding certain maximum value \( \delta_m \):

\[ \sin \delta_m = 1/(1 + 4N^2)^{1/2} \]  

(18)

If it was possible to use such kind of fiber with no external field-caused perturbations (wind fiber in coils with big radius, hold it with isotropic holders etc) that would mean that the matrix \( S \) would be determined by \( G = J_0 \). For that case \( \delta_m < 1/2N \),

\[ F_s = (1/4) <\text{Tr}\{S\}> > 1 - 1/2N, \]  

\[ F_s = <\text{Im}(S_{xx}) \text{Im}(S_{xy}) > <1/(4N^2). \]  

(19)

As soon as both accuracy and scale factor stability for the sensor are the better the bigger \( N \) is, for now the most popular kind of fiber for such sensors were those with the biggest achievable \( N \).

However it is not possible to find some simple sensor configuration protecting sensitive fiber from external fields. Also, it is a well-known fact that sensor’s sensitivity to current is proportional to the number of coils and not to fiber length itself. So, at least for current sensing it seems very attractive to find some robust option of sensitive fiber, which could allow fiber bending with radii small enough for industrial applications and in addition to allow less careful handling, holding etc.

From that point of view the high-\( N \) option of spun fiber is not optimal as long as its retardation is determined mainly by external birefringence especially bending-induced one. If we choose very low-\( N \) option of spun fiber then the sensor’s accuracy and scale factor stability will be worse (at least for
monochromatic lightsource) according to (19) and in the limit of N=0 we would also have in-fact high-birefringence fiber with zero sensitivity to magnetic field.

4. FIBER ROBUSTNESS

In order to evaluate optimal fiber parameters that could from one side let one have acceptable accuracy and sensitivity and from the other side to make fiber more robust one have to get some solution of equation (10) or (12) with non-zero perturbation $W$. We have to underline here that regular perturbation theory approximation for equation (12) will not allow us to achieve reliable result due to the following consideration. If maximum value of induced birefringence is $B$ then the sensitive fiber length for which perturbation theory-based results are valid, is less than $1/B$. For example, bending induced birefringence correspondent to reasonable radii of bending will have beatlength in the range 3 – 10 m and usually sensitive fiber length exceeds 30 m.

The most appropriate method for this task is so-called “method of averaging”. As we’ve already calculated the non-perturbed single-pass matrix $G$ has oscillating matrix components, their period of oscillation being $L_p/2$. Then if we want to obtain the main part of solution of (12) we have to express the matrix solution $X$ as: $X = X_0 + X_1$, where $X_1 << X_0$. Then we get for $X_0$:

$$i \frac{dX_0}{dz} = Av\{G^{-1} W G\} X_0 = w X_0$$

(20)

where $Av\{\ldots\}$ means averaging on “fast” coordinate (with periods $L_p/2$). The mentioned averaging one can simply understand as neglecting fast varying terms in $G^{-1} W G$. After completing this procedure one obtain $w$ components as:

$$w = \left(\frac{1}{2}\right) B_{ex}(z) \begin{bmatrix} \cos(2\Delta\kappa z) & \sin(2\Delta\kappa z) \\ \sin(2\Delta\kappa z) & -\cos(2\Delta\kappa z) \end{bmatrix}$$

(21)

$B_{ex}(z) = 2\pi/L_{ex}$ is external field-induced birefringence and $\tau = \Delta\kappa z = \kappa z - 2\pi z/L_p = [(1 + 1/4N^2)^{1/2} - 1] 2\pi z/L_p \approx 2\pi z/(4N^2L_p)$ is dimensionless variable.

Now we get two different cases, depending on the value of $\eta = B_{ex}(z)/\Delta\kappa \approx 4N^2 L_p/L_{ex}$. If $\eta > 1$ then the argument of oscillating terms is varying too slow to avoid external birefringence accumulating along the fiber length and this kind of fiber is not robust. If $\eta << 1$ then this oscillating term would not let perturbation term to accumulate and this fiber can stand some affectation.

$L_{ex} = \lambda/\Delta n$ can be evaluated for bending-induced case. For bended fused silica fibers $\Delta n \approx 0.133 (d/r)^2$ where $d$ and $r$ are fiber diameter and radius of bend. Then for 10-cm radius bend for 125 µm fiber we get for 800-nm range to be of the order of 5 m. If $L_p$ is about 5 mm then $\eta$ exceeds 1 for $N>16$ that is when intrinsic birefringence beatlength is more than 8 cm.

It seems to be reasonable to treat a spun fiber as robust one if the retardation caused by external-induced birefringence does not exceed (even for infinite fiber length) the maximum value of retardation caused by preform birefringence in accordance if (16). That means that for this definition of robustness parameter $\eta$ should not exceed $1/2N$. In other words, main robustness parameter $N$ has to be less than $N_{cr}$, which is determined by:

$$N_{cr} \approx (1/2) \left(\frac{L_{ex}}{L_p}\right)^{1/3}$$

(22)
5. RESULTS FOR BROADBAND LIGHTSOURCE OPTION

The above-derived equation (22) restricts the biggest number of \( N \), which will allow approximately perturbation-independent fiber behavior. However, for monochromatic lightsource this will also restrict sensor’s accuracy and scale-factor stability in accordance with equations from section 2:

\[
F_s = \frac{1}{4} \langle [\text{Tr}\{S\}] \text{Tr}\{s\} - \text{Tr}\{Ss\} \rangle \text{ Tr}\{S\} \rangle = 1 - \Delta F_s \sin^2[\Phi + \kappa z]
\]

\[
F_a = \langle \text{Im}(S_{sx}) \text{Im}(S_{sy}) \rangle = \Delta F_a \sin 2[\Phi + \kappa z]
\]  (23)

Here \( \Delta F_s, \Delta F_a \) are of the order of \( (1/2N)^2 \) and due to fast-oscillating terms in (23) both accuracy and scale factor vary strongly with external temperature, pressure etc changes. It seems that the most temperature dependent variable is \( N \) \( (dN/dT \sim 0.001) \), then for 100 degrees centigrade variation \( (\Delta T \sim 100) \) even for 10 m long sensitive fiber the harmonic term’s argument change exceeds \( 2\pi \). For most applications the sensitivity \( (1/2N)^2 \) reduction itself is acceptable however correspondent variations of both scale factor and additive error (drift) for \( N \sim 1 - 10 \) are not acceptable.

This situation could be improved\(^5\) for the broadband lightsource option when the sensitive fiber length is long enough to satisfy conditions:

\[
(\Delta \lambda / \lambda) L \kappa \gg 1, \quad (\Delta \lambda / \lambda) L \Delta \kappa \gg 1 \quad \text{ (24)}
\]

where \( \Delta \lambda \) is a source linewidth. Then as soon as the spatially-averaged double-pass matrix \( s \) appears to be wavelength-independent and oscillating terms of \( S \) vanish after bandwidth averaging due to (24), we get for \( F_s, F_a \) approximate expressions:

\[
F_s \approx 1 - 3(1/2N)^2 - 3\eta^2/2, \quad F_a = [(1/2N)^2 + \eta^2/2] \sin[2\alpha( L_{ex}) + 2\alpha l] \quad \text{ (25)}
\]

Additional \( L_{ex} \)-dependent angle \( \alpha( L_{ex}) \) appears due to perturbations and there are no fast oscillating terms in (25). That means that although scale factor difference from 1 and additive error are of the same \( (1/2N)^2 \) order to the monochromatic lightsource case, the temperature dependent variations with external parameters changes remain of the order of \( (dN/dT)\Delta T/(2N)^2 \), that is 10 times less than for monochromatic case. These results allow one to utilize sensitive fibers with \( N \) approximately 3 times less and that, in its turn, allow to have 30 times more strong external induced birefringence in accordance with (22) or 5 times less bending radius if main external birefringence is bending-induced.

6. DISCUSSION

The concept of robust low-birefringence (RLB) fibers for magnetic field or current sensor application was based on the Jones matrix calculation analysis, improved approximation for taking into account external field-induced perturbations being developed.

For any external field-induced birefringence the optimal fiber with robustness parameter \( N \) corresponding to derived equation of critical \( N_{cr} \approx (1/2) (L_{ex}/L_p)^{1/3} \) can be determined. The theoretical results proved RLB fibers to be most appropriate for practical sensor applications.
We are presently in the process of produced RLB fiber samples testing with N-parameter in the range: 1 - 20, some preliminary experimental results obtained up to now being in good accordance with theoretical predictions.

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8. REFERENCES


